

Local Gravity Disturbance Estimation from Multiple-High-Single-Low  
Satellite-To-Satellite Tracking

Christopher Jekeli

Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts

## ABSTRACT

The idea of satellite-to-satellite tracking in the high-low mode has received renewed attention in light of the uncertain future of NASA's proposed low-low mission, Geopotential Research Mission (GRM). The principal disadvantage with a high-low system is the increased time interval required to obtain global coverage since the intersatellite visibility is often obscured by Earth. The U.S. Air Force has begun to investigate high-low satellite-to-satellite tracking between the Global Positioning System (GPS) of satellites (high component) and NASA's Space Transportation System (STS), the shuttle (low component). Because the GPS satellites form, or will form, a constellation enabling continuous three-dimensional tracking of a low-altitude orbiter, there will be no data gaps due to lack of inter-visibility. Furthermore, all three components of the gravitation vector are estimable at altitude, a given grid of which gives a stronger estimate of gravity on Earth's surface than a similar grid of line-of-sight gravitation components. The proposed Air Force mission is STAGE (Shuttle-GPS Tracking for Anomalous Gravitation Estimation) and is designed for local gravity field determinations since the shuttle will likely not achieve polar orbits. The motivation for STAGE was the feasibility to obtain reasonable accuracies with absolutely minimal cost. Instead of simulating drag-free orbits, STAGE uses direct measurements of the nongravitational forces obtained by an inertial package onboard the shuttle. This paper analyzes the sort of accuracies that would be achievable from STAGE vis-a-vis other satellite tracking missions such as GRM and European Space Agency's POPSAT-GRM.

## 1. ASSUMPTIONS AND PARAMETERS

The observable in STAGE is the phase of the GPS carrier signal. It is differentiated twice to obtain the line-of-sight (LOS) acceleration of STS with respect to the GPS satellite. For the purpose of the analysis, it is assumed that this is the observed quantity and that it is a component of the difference between the gravity disturbance vectors at the two satellite locations. The actual difference between the LOS acceleration and a component of gravitation is insignificant on the average (see, e.g., Rummel 1980) and is therefore neglected here.

The error analysis is accomplished using the method of least-squares collocation which requires a covariance model for Earth's gravitational field (the Tscherning/Rapp model (Tscherning and Rapp, 1974) is used), the spatial coordinates of the data (sampled from Keplerian orbits), and a model for the noise of the data (assumed to be an uncorrelated process). Errors in the position of the satellites, though important, are not considered in this analysis.

Table 1 lists the adopted Keplerian elements of the satellites entering the analysis. The numbering of the GPS satellites is arbitrary. The two GRM satellites follow each other 300 km apart in the same orbit. Table 2 lists various (potential or fictitious) SST missions which possess the range of parameters to be considered in the analysis. The assumed acceleration accuracy of GRMa-GRMb (0.03 mgal) corresponds (according to an algorithm developed by Rummel (1980)) to the actual range-rate observational accuracy of  $10^{-6}$  m/s (Keating et al., 1986); whereas GRM-POPSAT's assumed 0.7 mgal acceleration accuracy corresponds to  $25 \times 10^{-6}$  m/s (Reigber et al., 1987). All GPS tracking missions have an assumed accuracy in acceleration of 1 mgal for a 75 s integration time. The difference between STS-GPS#1 and STS-GPS#6 is the zenith angle of the LOS; with GPS#1 (#6) it is generally greater (less) than  $45^\circ$ . The designation nGPS means that the full 18-satellite configuration of GPS is used, but only three satellites track the low orbiter at a time. The three chosen satellites have the greatest degree of mutual orthogonality of the LOS vectors. In order to obtain somewhat comparable data distributions, a sampling interval of 75 s was chosen for each mission.

Table 1: SST satellites and their Keplerian elements ( $e=0$ ,  $\omega=0$ ).

Keplerian Elements	STS	POPSAT	GPS#1	GPS#6	GRMa	GRMb
Altitude [km]	300.	7000.	20189.	20189.	160.	160.
Inclination [deg.]	28.5	98.	55.	55.	90.	90.
R.A. of Asc. Node [deg.]	45.	270.	0.	60.	90.	90.
Time of Perigee Pass. [s]	0.	0.	0.	-33505.	0.	-38.4

Table 2: SST missions and parameters defining resolution at altitude.

Parameters	GRMa-GRMb	GRM-POPSAT	STS-GPS#1	STS-GPS#6	STS-nGPS	GRM-nGPS
Int. Time [s]	4	10	75	75	75	75
Accur. [mgal]	0.03	0.7	1.	1.	1.	1.

The data points are limited to a square region symmetric about the equator and zero meridian. Only those points are included where the zenith angle to the high satellite is less than  $100^\circ$ . The estimated quantity is the  $2^\circ$ -mean gravity disturbance on Earth's surface. The error is estimated for a total of nine such quantities at coordinates in latitude and longitude:  $(\pm 2^\circ, \pm 2^\circ)$ ,  $(\pm 2^\circ, 0^\circ)$ ,  $(0^\circ, \pm 2^\circ)$ ,  $(0^\circ, 0^\circ)$ . The error curves shown in the next section represent the root-mean-square (RMS) of these nine error estimates.

## 2. RESULTS

Figure 1 shows the RMS estimated error in  $2^\circ$ -mean gravity disturbances as a function of data density for the missions of Table 2. The data area is a  $10^\circ \times 10^\circ$  square (hence a data density of 1 means that it contains 100 points more or less randomly distributed). Since the vertical component of the gravity disturbance is more highly correlated with itself than with the horizontal components, the error with STS-GPS#6 is much less than with STS-GPS#1; and a low-low mission is quite poor in comparison. For similar reasons, there is some, but not an overwhelming improvement in observing all three components of the gravity disturbance vector at altitude (STS-

nGPS). Major improvement comes by reducing the altitude (GRM-nGPS). The low errors of GRM-POPSAT arise from a combination of favorable factors: low altitude, short integration time, low data noise, and complementary orbital parameters.

In Figure 2, the RMS error is a function of data extent. For the high-low STS missions, little is gained by extending the data area beyond a certain size. Because of the longer correlation length of the horizontal gravity disturbance, a wider area is required for the low-low mission.

The instability of the GPS clock frequency dominates the data noise for the GPS tracking missions. It is assumed that all errors in the error budget not associated with this instability have a combined standard deviation of 0.5 mgal. The clock instability is characterized by the Allan variance which is often modelled as inversely proportional to the integration time. It is assumed here that the acceleration noise due to this instability is proportional to the Allan standard deviation divided by the integration time (Upadhyay et al., 1988). By monitoring the short-term fluctuations of these clocks at ground tracking stations having more stable clocks, Upadhyay et al. (1988) estimate that a hundred-fold improvement in the Allan variance can potentially be achieved. Figure 3 shows the RMS error of estimation as a function of integration time for the two multiple-high-single-low SST missions. Increasing the integration time decreases the data noise, but more of the shorter wavelengths of the gravity field are obliterated. Conversely, as the integration time decreases, the observations are more sensitive to the short-term fluctuations in the gravity

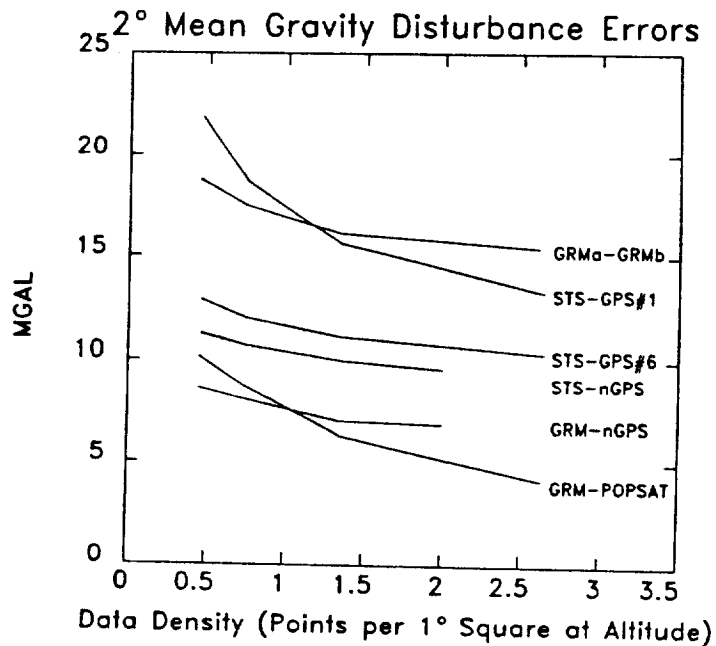


Figure 1

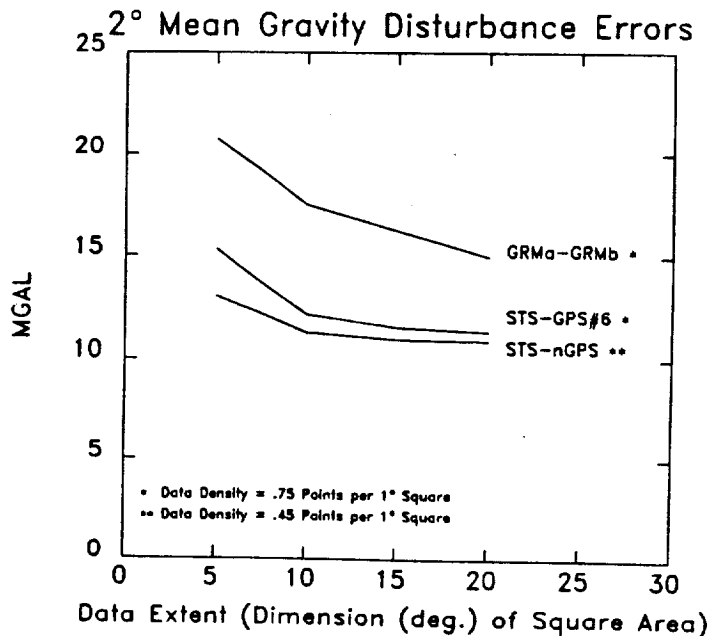


Figure 2

field, but the signal-to-noise ratio is smaller. Therefore, there is a definite optimum integration time for a given Allan variance. An improvement by a factor of 10 in the Allan variance gives a total data noise of 1 mgal at 75 s integration time. A 90-fold improvement implies a total data noise of 1 mgal with a 37 s integration time. The optimal integration time also depends on the altitude of the low orbiter.

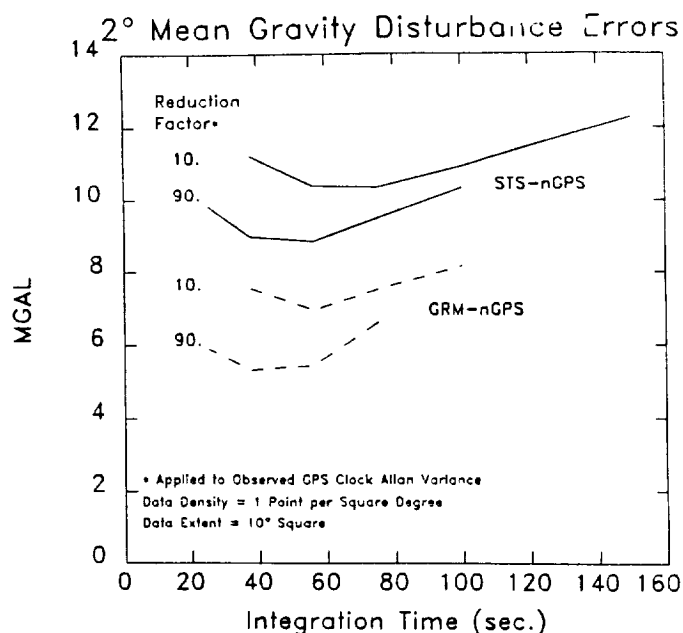


Figure 3

### 3. CONCLUSION

Although not as accurate as proposed dedicated gravity mapping missions, satellite-to-satellite tracking using the GPS can contribute to an improvement of present models of Earth's gravity. Even STAGE, to be viewed as a demonstration of the concept, would improve the model locally over land areas, such as parts of Asia, Africa, and South America.

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